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SCATTERING FROM A RANDOM SURFACE

By: H.D.I. Abarbanel

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In this note we will give a novel formulation of the scattering of waves from a random surface. This problem, both for acoustic waves and electromagnetic waves, has a long and well documented literature. Our contribution will be a recasting of the problem into a form which automatically meets the boundary conditions on the random surface and on the other boundaries at the expense of adding to the usual wave equation a fluctuation operator which depends on derivatives of the surface with respect to space and time.

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ABSTRACT

THIS REPORT ~~We~~ gives a formulation of the problem of propagation of scalar waves over a random surface. By a judicious choice of variables IT IS ~~we are~~ able to show that this situation is equivalent to propagation of these waves through a medium of random fluctuations with fluctuating source and receiver. The wave equation in the new co-ordinates has an additional term, the fluctuation operator, which depends on derivatives of the surface in space and time. An expansion in the fluctuation operator is given which guarantees the desired boundary conditions at every order. THIS REPORT ~~We~~ treat both the cases where the surface is time dependent, such as the sea or surface, or fixed in time. Also discussed is the situation where the source and receiver lie between the random surface and another, possibly also random, surface. In detail THIS REPORT ~~we~~ considers acoustic waves for which the surfaces are pressure release. The method is directly applicable to electromagnetic waves and other boundary conditions.

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I INTRODUCTION

In this note we will give a novel formulation of the scattering of waves from a random surface. This problem, both for acoustic waves and electromagnetic waves, has a long and well documented literature[1,2]. Our contribution will be a recasting of the problem into a form which automatically meets the boundary conditions on the random surface and on the other boundaries at the expense of adding to the usual wave equation a fluctuation operator which depends on derivatives of the surface with respect to space and time.

To state our observation, let us suppose there is a source of scalar waves $\psi(\vec{x},t)$ above a random surface located at $z = \zeta(\underset{\sim}{x},t)$, $\underset{\sim}{x} = (x,y)$. The difficulty in solving the wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{x},t) = \text{source} \tag{1}$$

in the presence of the random surface comes in meeting the boundary conditions on that surface. Constructing a Green function in closed form for the wave operator with an irregularly shaped surface is likely impossible. Many authors[2] seem to make some kind of approximation to the surface which allows the determination of a Green function over the approximate surface and then enforce the boundary condition as a Taylor

series in the r.m.s. value of the variable $\zeta(\underset{\sim}{x},t)$ - approximate surface . If the approximate surface is $z = 0$, as is reasonable when $\langle\zeta(\underset{\sim}{x},t)\rangle = 0$, then a small wave height expansion in the r.m.s. value of ζ is employed.

We observe here that if one transforms to a new co-ordinate system

$$\xi = z - \zeta(\underset{\sim}{x},t) \ , \tag{2}$$

$$\underset{\sim}{\rho} = \underset{\sim}{x} \ , \tag{3}$$

and

$$\tau = t \ , \tag{4}$$

then in the ξ , $\underset{\sim}{\rho}$, τ co-ordinates the boundary condition is on the plane $\xi = 0$, and it is rather easy to construct the Green function for the wave equation meeting the chosen condition. Two features now appear in this co-ordinate choice, and the complications of the problem lie in them: 1) the source and receiver of the waves are moving in a random fashion. This is actually not a serious complication as we shall see. 2) The wave equation is transformed into a stochastic equation

$$\left[\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\rho_j^2} - \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2} - F(\xi,\underset{\sim}{\rho},\tau;\zeta)\right] \Psi(\xi,\underset{\sim}{\rho},\tau) = \text{source}, \qquad j=1,2 \quad , \tag{5}$$

where the <u>fluctuation operator</u> F is a differential operator depending on derivatives of ζ . The construction of the Green function for the stochastic wave operator in (5) is, of course, tantamount to the original difficult problem. However, we now see a natural perturbation series emerging in the form of an expansion in the fluctuation operator. Since F depends only on derivatives of ζ , one might hope that for a surface

smooth enough in space and time, the series would converge rapidly.

Once the problem has been transformed into (5) it is seen to be the same as scalar wave propagation through a random medium[3] with the additional twist here of randomly moving sources. The literature on propagation in a random medium is extensive and it may be that techniques developed in those studies will carry over here as well[4,5].

In this note we address the most rudimentary problems. We consider a source of scalar waves located at some point z_o, $\underset{\sim}{x}_o$ above a random surface. Its time dependence is simply $s(t)$, which could be just $e^{-i\omega t}$ for a monochromatic transmitter. We have in mind sound waves propagating in the ocean with $\zeta(\underset{\sim}{x},t)$ being the sea surface at which we shall choose the pressure $\psi(z,\underset{\sim}{x},t)$ to vanish

$$\psi\Big(\zeta(\underset{\sim}{x},t),\underset{\sim}{x},t\Big) = 0 \quad . \tag{6}$$

Our techniques will certainly apply to the scattering of electromagnetic waves with appropriate boundary conditions, but we do not consider them here.

The discussion covers four cases. First we make the so-called "narrow band approximation"[2] which treats the surface as slowly varying during many cycles of the source. The wave equation then becomes the Helmholtz equation when the source is $s(t) = e^{-i\omega t}$. In this <u>slow surface approximation</u>, we consider scattering without a "bottom"; that is,

with an infinite half-space above $\zeta(\underset{\sim}{x},t)$. Then we consider a "bottom" at $z = B$ with the source located in $\zeta(\underset{\sim}{x},t) \leq z \leq B$ and $\psi(B,\underset{\sim}{x},t) = 0$ also. One could make the bottom be random in space as well, say, at $z = B(\underset{\sim}{x})$, but we have not studied this.

Next we treat the full problem with a time dependent surface both without a bottom and with. This is conceptually just as easy as the slow surface situation but involves a bit more algebra.

The basic result will be a form for $\psi(z,\underset{\sim}{x},t)$ which has a natural expansion in the fluctuation operator F . Averaging over realizations of $\zeta(\underset{\sim}{x},t)$ is possible term by term in the series in F when ζ has a gaussian distribution. There is, as usual[3], a diagramatic representation of that series. We have not yet explored this in any detail, except to note its resemblance to similar series in quantum field theory. It may be that techniques of quantum field theory for summing infinite subsets of similar series will be found useful here as well.

II FORMULATION OF THE PROBLEM AND THE SLOW SURFACE APPROXIMATION

We want to consider a medium with a constant sound speed lying above a surface $z = \zeta(\underset{\sim}{x},t)$. We are interested in the sound pressure $\psi(z,\underset{\sim}{x},t)$ received at $z,\underset{\sim}{x},t$ from a source located at z_o, $\underset{\sim}{x}_o$. The wave equation reads

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(z,\underset{\sim}{x},t) = s(t)\delta(z-z_o)\delta^2(\underset{\sim}{x}-\underset{\sim}{x}_o) \quad , \qquad (7)$$

and we wish to require that $\zeta(\underset{\sim}{x},t)$ be a pressure release surface

$$\psi\left(\zeta(\underset{\sim}{x},t),\ \underset{\sim}{x},\ t\right) = 0 \qquad (8)$$

as well as asking that as $z^2 + \underset{\sim}{x}^2 \to \infty$, $\psi(z,\underset{\sim}{x},t) \to 0$. (See Fig. 1.)

Later we return to the full problem just given. Now we take $s(t) = e^{-i\omega t}$ and assume that the motions in $\zeta(\underset{\sim}{x},t)$ are slow compared to $e^{-i\omega t}$. As shown in Ref. 2 this leads us to consider the Helmholtz equation $(k = \omega/c)$.

$$(\nabla^2 + k^2)\ \psi(z,\underset{\sim}{x}) = \delta(z-z_o)\delta^2(\underset{\sim}{x}-\underset{\sim}{x}_o) \qquad (9)$$

with the boundary conditions

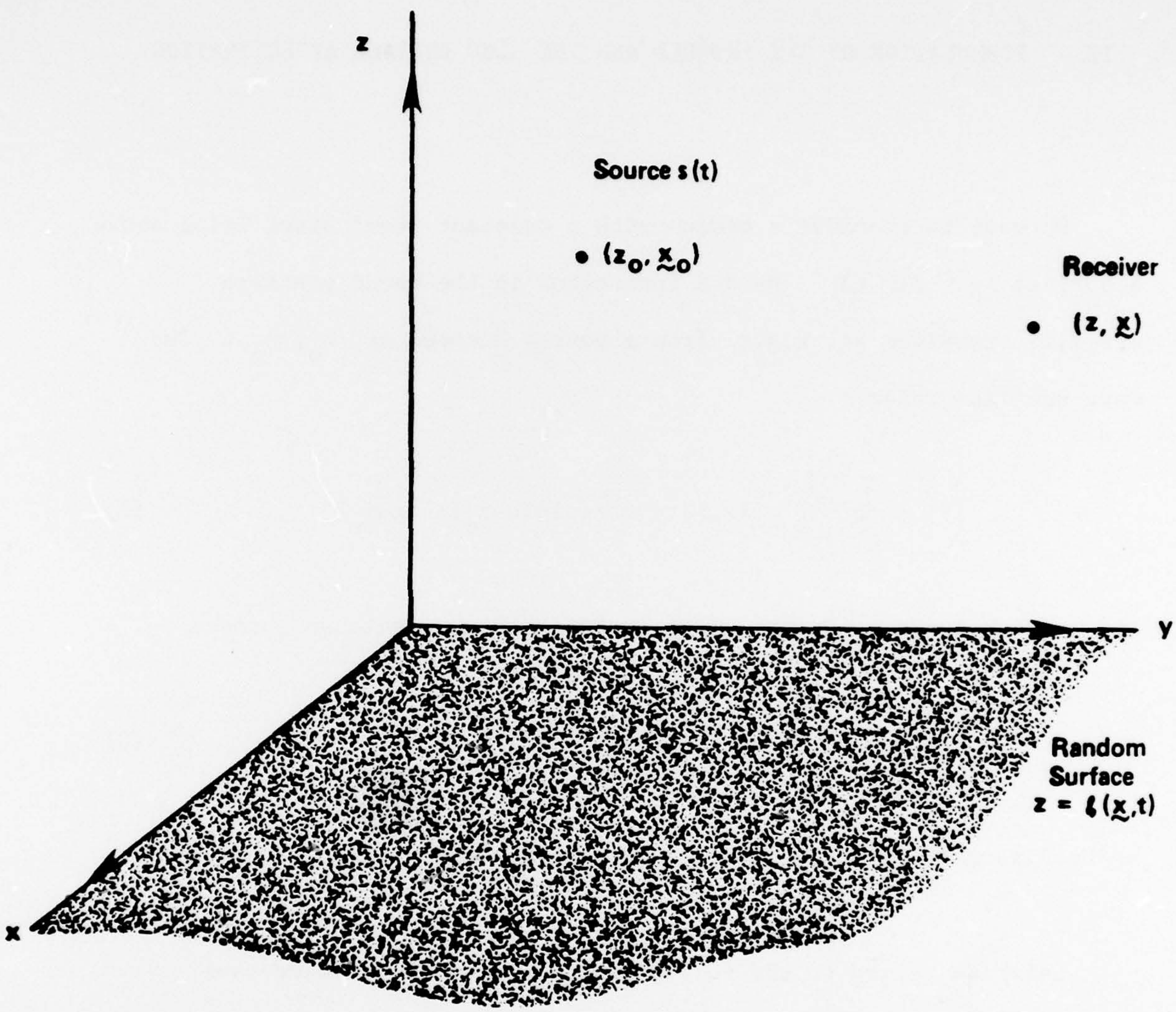

Figure 1 THE GEOMETRICAL SITUATION CONSIDERED IN THIS PAPER

A source s(t) at $(z_0, \underset{\sim}{x}_0)$ ($\underset{\sim}{x}_0 = (x_0, y_0)$) is located above a random surface $z = \zeta(\underset{\sim}{x}, t)$. We want to know the acoustic pressure field $\psi\ (z, \underset{\sim}{x}, t)$ when the pressure is required to vanish on the random surface and at infinity.

$$\psi\big(\zeta(\underset{\sim}{x}) , \underset{\sim}{x}\big) = 0 \quad . \tag{10}$$

and
$$\psi(z,\underset{\sim}{x}) \to 0 \qquad z^2 + \underset{\sim}{x}^2 \to \infty \quad . \tag{11}$$

The time dependence of both ψ and ζ will be suppressed now as they play an entirely inessential role in the development. At any point desired the reader may restore the slow time variation in ζ and recall the $e^{i\omega t}$ multiplying $\psi(\underset{\sim}{x})$. The problem as presently formulated is shown in Fig. 2. It will be called the slow surface approximation.

Now we perform the change of variables indicated in the introduction

$$\xi = z - \zeta(\underset{\sim}{x}) \quad , \tag{12}$$

and
$$\underset{\sim}{\rho} = \underset{\sim}{x} \quad , \tag{13}$$

which maps the random surface to the plane $\xi = 0$. In this co-ordinate system the boundary conditions on the pressure field $\Psi(\xi,\underset{\sim}{\rho}) = \psi(z,\underset{\sim}{x})$ read

$$\Psi(0,\underset{\sim}{\rho}) = 0 \quad , \tag{14}$$

and
$$\Psi(\xi,\underset{\sim}{\rho}) \to 0 \qquad \xi^2 + \underset{\sim}{\rho}^2 \to \infty \quad . \tag{15}$$

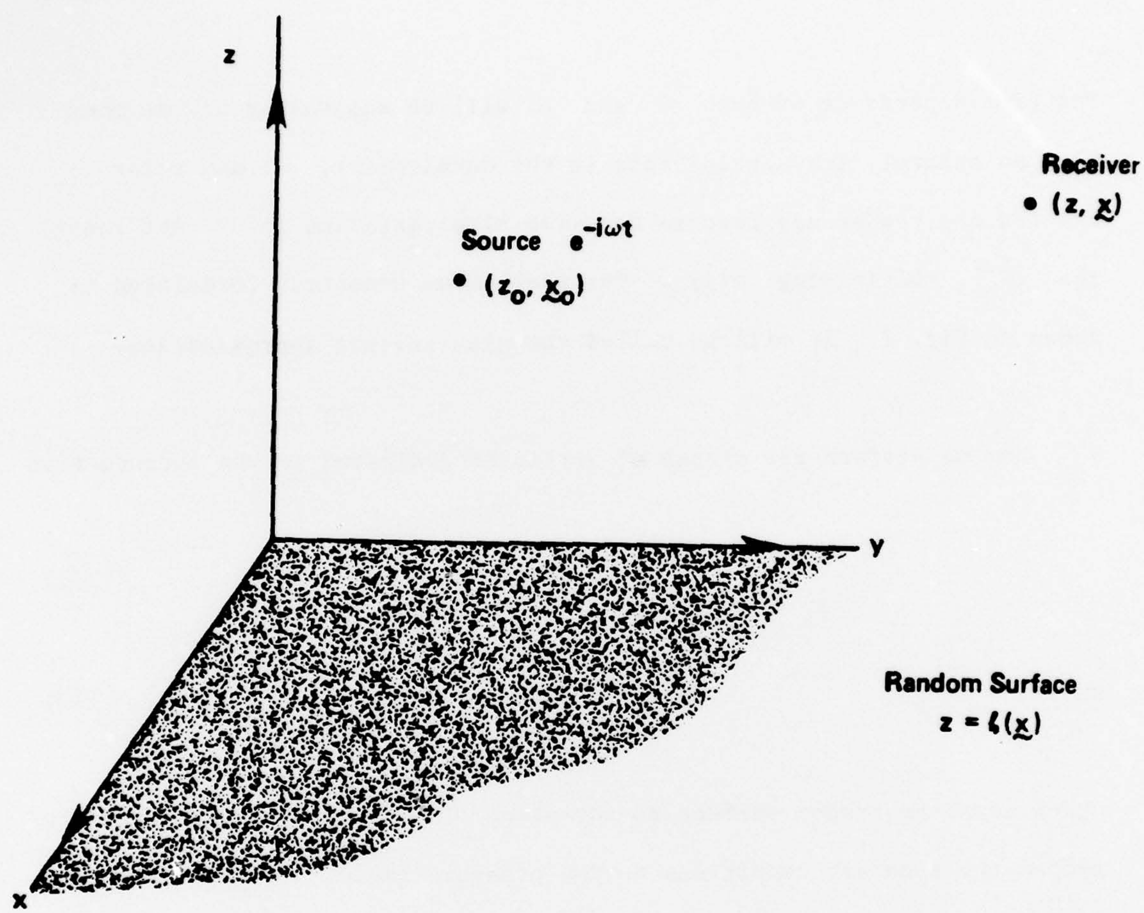

Figure 2 The same situation as in Figure 1 but the source has been taken to be mono-chromatic, $s(t) = e^{-i\omega t}$, and the slow surface approximation has been made. This means the dominant frequencies in $\zeta(x, t)$ are much less than ω.

These boundary conditions will be easy to meet always. The source and the receiver are now located at points that randomly fluctuate about z_o and z with the statistics of $\zeta(\underset{\sim}{x})$. See Figure 3.

The complication of the problem resides in the wave equation. Using the chain rule we learn

$$\frac{\partial\Psi(\xi,\underset{\sim}{\rho})}{\partial z} = \frac{\partial\Psi(\xi,\underset{\sim}{\rho})}{\partial\xi} \quad , \tag{16}$$

and

$$\frac{\partial}{\partial x_j}\Psi(\xi,\underset{\sim}{\rho}) = \frac{\partial}{\partial\rho_j}\Psi(\xi,\underset{\sim}{\rho}) - \frac{\partial\zeta(\underset{\sim}{\rho})}{\partial\rho_j}\,\frac{\partial\Psi(\xi,\underset{\sim}{\rho})}{\partial\xi} \qquad j=1,2 \quad . \tag{17}$$

This transforms the wave equation (9) into

$$\left(\frac{\partial^2}{\partial\xi^2} + \nabla_\perp^2 + k^2 - F(\xi,\underset{\sim}{\rho})\right)\Psi(\xi,\underset{\sim}{\rho}) = \delta\left(\xi-\left(z_o - \zeta(\underset{\sim}{\rho}_o)\right)\right)\,\delta^2(\underset{\sim}{\rho}-\underset{\sim}{\rho}_o) \tag{18}$$

with

$$\nabla_\perp = \left(\frac{\partial}{\partial\rho_1} , \frac{\partial}{\partial\rho_2}\right) \tag{19}$$

,

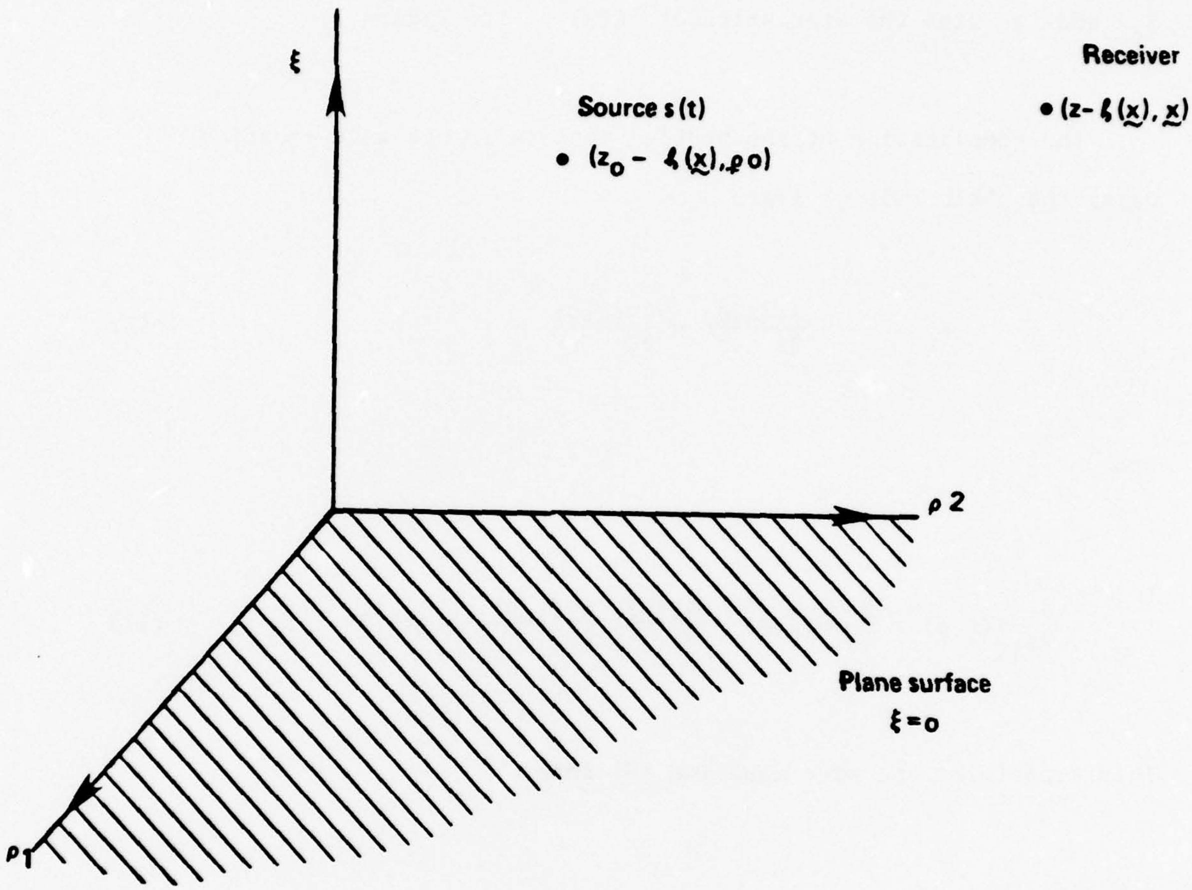

Figure 3 THE MAPPED SCATTERING PROBLEM IN THE SLOW SURFACE APPROXIMATION

The source and receiver move randomly in the ξ direction, but the boundary surface is now the plane $\xi = o$ and the surface at infinity. The wave operator for the field $\Psi(\xi, \rho)$ contains a fluctuation operator. By choosing the Green function in the absence of this fluctuation operator to match the boundary conditions desired, further approximations including the fluctuation operator always satisfy the boundary conditions.

the gradient operator in the ρ direction, and $F(\xi,\rho)$ is the <u>fluctuation operator</u>

$$F(\xi,\rho) = -(\nabla_\perp\zeta)^2 \frac{\partial^2}{\partial\xi^2} + (\nabla_\perp^2\zeta)\frac{\partial}{\partial\xi} + 2(\nabla_\perp\zeta)_j \frac{\partial}{\partial\rho_j}\frac{\partial}{\partial\xi} \quad . \tag{20}$$

The solution to this Helmholtz equation is given as usual in terms of some Green function $G_o(\xi, \rho ; \xi', \rho')$ satisfying

$$\left(\frac{\partial^2}{\partial\xi^2} + \nabla_\perp^2 + k^2\right) G_o(\xi, \rho ; \xi', \rho') = \delta(\xi-\xi')\delta(\rho-\rho') \tag{21}$$

in the half space $\xi,\xi' \geq 0$. If we call

$$\mathcal{J}(\xi,\rho) = F(\xi,\rho)\Psi(\xi,\rho) + \delta\Big(\xi-\big(z_o - \zeta(\rho_o)\big)\Big)\,\delta(\rho-\rho_o) \quad , \tag{22}$$

the solution reads[6]

$$\Psi(\xi,\rho) = \int_o^\infty d\xi' \int d^2\rho'\, G_o(\xi,\rho;\, \xi'\, \rho')\mathcal{J}(\xi'\, \rho')$$

$$+ \int_{\text{surface}} dS\, \hat{n}. \Big[\Psi\big(\xi_s,\rho_s\big)\nabla_s G\big(\xi,\rho;\, \xi_s,\rho_s\big) - \nabla_s\Psi\big(\xi_s,\rho_s\big) G\big(\xi,\rho;\, \xi_s,\rho_s\big)\Big] \tag{23}$$

where dS is over the surface $\xi = 0$ and the boundary at infinity above it, $\xi_s, \underset{\sim}{\rho}_s$ are co-ordinates in the surface; ∇_s is the gradient on the surface; and $\hat{n}$ points out of the volume enclosed by the surface.

We will choose a G_o which vanishes on the surface, so the second term in (23) is absent. This G_o is constructed by putting a negative image source at $(-\xi', \underset{\sim}{\rho}')$ to match the source in (21):

$$G_o(\xi, \underset{\sim}{\rho}\, ; \xi', \underset{\sim}{\rho}') = \int \frac{d^3Q}{(2\pi)^3} \frac{e^{i\underset{\sim}{Q} \cdot (\underset{\sim}{\rho}-\underset{\sim}{\rho}')}}{k^2-\underset{\sim}{Q}^2-Q_3^2 + i\varepsilon} \left[e^{iQ_3(\xi-\xi')} - e^{iQ_3(\xi+\xi')} \right], \quad (24)$$

$$= -\frac{1}{4\pi} \left(\frac{e^{ikR_+}}{R_+} - \frac{e^{ikR_-}}{R_-} \right) , \quad (25)$$

with

$$R_{\pm} = \left((\xi \mp \xi')^2 + (\underset{\sim}{\rho}-\underset{\sim}{\rho}')^2 \right)^{\frac{1}{2}} . \quad (26)$$

Now (22) becomes an integral equation for Ψ

$$\Psi(\xi, \underset{\sim}{\rho}) = G_o\left(\xi, \underset{\sim}{\rho};\ z_o - \zeta(\underset{\sim}{\rho}_o),\ \underset{\sim}{\rho}_o\right)$$

$$+ \int_0^{\infty} d\xi' \int d^2\rho'\ G_o(\xi, \underset{\sim}{\rho};\ \xi', \underset{\sim}{\rho}') F(\xi', \underset{\sim}{\rho}') \Psi(\xi', \underset{\sim}{\rho}') \quad . \quad (27)$$

The formal solution to this equation is most easily written in operator form. Let Ψ be a vector in $\xi, \underset{\sim}{\rho}$ space and G_o and F operators.

$$\Psi = G_o S + G_o F \Psi \tag{28}$$

with S the operator with $\xi, \underset{\sim}{\rho}$ representation

$$S = \delta\Big(\xi - \big(z_o - \zeta(\underset{\sim}{\rho})\big)\Big)\, \delta^2(\underset{\sim}{\rho} - \underset{\sim}{\rho}_o) \quad . \tag{29}$$

Ψ is given formally by

$$\Psi = G\, S \quad , \tag{30}$$

with G the full Green operator

$$G = \frac{1}{1 - G_o F}\, G_o = G_o\, \frac{1}{1 - F G_o} \quad . \tag{31}$$

Slightly more specifically

$$\Psi(\xi, \underset{\sim}{\rho}) = G\Big(\xi, \underset{\sim}{\rho};\;\; z_o - \zeta(\underset{\sim}{\rho}_o), \underset{\sim}{\rho}_o\Big) \quad , \tag{32}$$

and

$$\psi(z, \underset{\sim}{x}) = G\Big(z - \zeta(\underset{\sim}{x}),\ \underset{\sim}{x};\ z_o - \zeta(\underset{\sim}{\rho}_o),\ \rho_o\Big) \quad . \tag{33}$$

Any approximation to G , for example expansion of (31) in F , will give $\psi\big(z = \zeta(\underset{\sim}{x}), \underset{\sim}{x}\big) = 0$ since G_o obeys that.

We have not investigated in any detail the properties of the operator G. It is clear that finding it is tantamount to finding the Green function for the propagation of scalar waves in a random medium where the "medium" is characterized by the fluctuation operator. There is a difference here which makes the present problem richer; namely, the source at $z_o - \zeta(\underset{\sim}{x}_o)$, $\underset{\sim}{x}_o$ and the receiver at $z-\zeta(\underset{\sim}{x})$, $\underset{\sim}{x}$ also move about randomly, so averaging $\psi(z,\underset{\sim}{x})$ over motions of ζ involves a bit more labor than in the usual random medium example where only the analogue of F fluctuates.

To isolate this last feature we rewrite G as follows

$$G = \left(\frac{1}{1-G_oF} - 1 + 1\right) G_o = G_o + \frac{1}{1-G_oF} G_oFG_o \tag{34}$$

$$= G_o + G_oF \frac{1}{1-G_oF} G_o \tag{35}$$

and

$$G = G_o\left(\frac{1}{1-FG_o} - 1 + 1\right) = G_o + G_o \frac{1}{1-FG_o} FG_o \tag{36}$$

Average these two forms of G for symmetry, and we have

$$G = G_o + G_o\mathcal{M}G_o \tag{37}$$

with $\mathcal{M}$ the operator

$$\mathcal{M} = \frac{1}{2}\left(F \frac{1}{1-G_o F} + \frac{1}{1-FG_o} F\right) \quad . \tag{38}$$

Now the function $G\left(z-\zeta(\underset{\sim}{x}), \underset{\sim}{x};\ z_o-\zeta(\underset{\sim}{x}_o), \underset{\sim}{x}_o\right)$ we seek can be written

$$G\left(z-\zeta(\underset{\sim}{x}), \underset{\sim}{x};\ z_o-\zeta(\underset{\sim}{x}_o), \underset{\sim}{x}_o\right)$$

$$= G_o\left(z-\zeta(\underset{\sim}{x}), \underset{\sim}{x};\ z_o-\zeta(\underset{\sim}{x}_o), \underset{\sim}{x}_o\right)$$

$$+ \int d\xi_1 d^2\rho_1 d\xi_2 d^2\rho_2\ G_o\left(z-\zeta(\underset{\sim}{x}), \underset{\sim}{x};\ \xi_1, \rho_1\right)$$

$$\mathcal{M}(\xi_1, \rho_1;\ \xi_2, \rho_2)\ G_o\left(\xi_2, \rho_2;\ z_o-\zeta(\underset{\sim}{x}_o), \underset{\sim}{x}_o\right) \tag{39}$$

Using the explicit form for G_o given in (24) we can place the troublesome $\zeta(\underset{\sim}{x}_o)$ and $\zeta(\underset{\sim}{x})$, the fluctuating parts of the source and receiver, in the exponential, so the averages over variations in ζ may be carried out.

Consider to this end the first term in (39). It reads

$$G_o\left(z-\zeta(\underset{\sim}{x}), \underset{\sim}{x};\ z_o-\zeta(\underset{\sim}{x}_o), \underset{\sim}{x}_o\right)$$

$$= \int \frac{d^3Q}{(2\pi)^3} \frac{e^{i\underset{\sim}{Q}(\underset{\sim}{x}-\underset{\sim}{x}_o)}}{k^2-Q^2-Q_3^2 + i\varepsilon} \left[e^{iQ_3(z-z_o)} e^{-iQ_3\left(\zeta(\underset{\sim}{x})-\zeta(\underset{\sim}{x}_o)\right)} - e^{iQ_3(z+z_o)} e^{iQ_3\left(\zeta(\underset{\sim}{x}) + \zeta(\underset{\sim}{x}_o)\right)}\right] \quad , \tag{40}$$

and we wish to average this over the probability distribution functional for $\zeta(\mathbf{u})$, call it $P[\zeta(\mathbf{u})]$.

For this purpose we need the characteristic functional $Z[J(\mathbf{u})]$ of $P[\zeta(\mathbf{u})]$,

$$Z[J(\mathbf{u})] = \int \prod_{\mathbf{u}} d\zeta(\mathbf{u}) \; P[\zeta(\mathbf{u})] \exp \int d^2 w J(\mathbf{w}) \zeta(\mathbf{w}) \tag{41}$$

and then the first term in the average of (40) requires us to choose $J(\mathbf{u}) = -iQ_3 \delta^2(\mathbf{u}-\mathbf{x}) + iQ_3 \delta^2(\mathbf{u}-\mathbf{x}_o)$ and in the second term choose $J(\mathbf{u}) = -\, iQ_3 \left(\delta^2(\mathbf{u}-\mathbf{x}) + \delta^2(\mathbf{u}-\mathbf{x}_o)\right)$.

To be more concrete, suppose the random surface is homogenous in $\mathbf{x}$ and gaussian with zero mean and correlation function

$$\langle \zeta(\mathbf{u}) \zeta(\mathbf{w}) \rangle = \Gamma(\mathbf{u}-\mathbf{w}) = \Gamma(\mathbf{w}-\mathbf{u}) \quad , \tag{42}$$

then

$$P[\zeta(\mathbf{u})] = N \exp - \frac{1}{2} \int d^2 u \, d^2 w \, \zeta(\mathbf{u}) \Gamma^{-1}(\mathbf{u}-\mathbf{w}) \zeta(\mathbf{w}) \tag{43}$$

with N a normalization factor. $Z[J(\mathbf{u})]$ is

$$Z[J(\mathbf{u})] = \exp \frac{1}{2} \int d\mathbf{u} \, d\mathbf{w} \, J(\mathbf{u}) \Gamma(\mathbf{u}-\mathbf{w}) J(\mathbf{w}) \quad . \tag{44}$$

Under these circumstances, the average of (40) is

$$\left\langle G_o\left(z-\zeta(\underset{\sim}{x}),\ \underset{\sim}{x};\ z_o-\zeta(\underset{\sim}{x}_o),\ \underset{\sim}{x}_o\right)\right\rangle$$

$$= \int \frac{d^3Q}{(2\pi)^3} \frac{e^{i\underset{\sim}{Q}(\underset{\sim}{x}-\underset{\sim}{x}_o)}}{k^2-\underset{\sim}{Q}^2-Q_3^2+i\varepsilon} \left\{ e^{iQ_3(z-z_o)} \left\langle e^{-iQ_3\left(\zeta(\underset{\sim}{x})-\zeta(\underset{\sim}{x}_o)\right)} \right\rangle \right.$$

$$\left. - e^{iQ_3(z+z_o)} \left\langle e^{-iQ_3\left(\zeta(\underset{\sim}{x}) + \zeta(\underset{\sim}{x}_o)\right)} \right\rangle \right\} \tag{45}$$

$$= \int \frac{d^3Q}{(2\pi)^3} \frac{e^{i\underset{\sim}{Q}(\underset{\sim}{x}-\underset{\sim}{x}_o)}}{k^2-\underset{\sim}{Q}^2-Q_3^2+i\varepsilon} \left\{ e^{iQ_3(z-z_o)} e^{-Q_3^2\left(\Gamma(0)-\Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)\right)} \right.$$

$$\left. - e^{iQ_3(z+z_o)} e^{-Q_3^2\left(\Gamma(0) + \Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)\right)} \right\} \quad . \tag{46}$$

Since $\Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o) \leq \Gamma(0)$ and $\Gamma(0) \geq 0$ for any physical surface $\zeta(\underset{\sim}{x})$, these integrals are well defined.

We can carry out most of the integrals in (46) by introducing the parameter λ via

$$\frac{1}{k^2-\underset{\sim}{Q}^2-Q_3^2+i\varepsilon} = -i \int_0^\infty d\lambda e^{i\lambda\left(k^2-\underset{\sim}{Q}^2-Q_3^2+i\varepsilon\right)} \quad , \tag{47}$$

and performing the indicated gaussian integrals. This leads to

$$\left\langle G_o\left(z-\zeta(\underset{\sim}{x}),\ \underset{\sim}{x};\ z_o-\zeta(\underset{\sim}{x}_o),\ \underset{\sim}{x}_o\right)\right\rangle$$

$$= -\frac{1}{8\pi^{3/2}} \int_0^\infty \frac{d\lambda}{\lambda}\, e^{i\lambda k^2} \exp + i(\underset{\sim}{x}-\underset{\sim}{x}_o)^2/4\lambda$$

$$\left\{ \frac{1}{\left[\left(\Gamma(0)-\Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)\right) + i\lambda\right]^{\frac{1}{2}}} \exp\left[-(z-z_o)^2/4\left(\Gamma(0)-\Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o) + i\lambda\right)\right]\right.$$

$$\left. - \frac{\exp\left[-(z+z_o)^2/4\left(\Gamma(0) + \Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o) + i\lambda\right)\right]}{\left[\Gamma(0) + \Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o) + i\lambda\right]^{\frac{1}{2}}} \right\} . \tag{48}$$

Various approximations suggest themselves to this moderately simple integral representation for the average pressure. For example, if k^2 is "large" then $\lambda=0$ will dominate the integral and we may neglect it relative to $\Gamma(0) \pm \Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)$. Then the λ integral gives $\pi i\, H_o^{(1)}\left(k\sqrt{(\underset{\sim}{x}-\underset{\sim}{x}_o)^2}\right)$; or since k is large, we have in that approximation

$$\left\langle G_o\left(z-\zeta(\underset{\sim}{x}),\ \underset{\sim}{x};\ z_o-\zeta(\underset{\sim}{x}_o),\ \underset{\sim}{x}_o\right)\right\rangle$$

$$\approx -\sqrt{\frac{2i}{kR}}\ \frac{e^{ikR}}{8\pi} \left\{ \frac{\exp\left[-(z-z_o)^2/4\left(\Gamma(0)-\Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)\right)\right]}{\left[\Gamma(0)-\Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)\right]^{\frac{1}{2}}}\right.$$

$$\left. - \frac{\exp\left[-(z+z_o)^2/4\left(\Gamma(0) + \Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)\right)\right]}{\left[\Gamma(0) + \Gamma(\underset{\sim}{x}-\underset{\sim}{x}_o)\right]^{\frac{1}{2}}} \right\} , \tag{49}$$

with $R^2 = (\underset{\sim}{x}-\underset{\sim}{x}_o)^2$.

The evaluation of the next term in G might rely on some useful approximation to the operator $\mathcal{M}$ based on detailed properties of F, that is of $\zeta(\underset{\sim}{x})$ and its derivatives, or one might simply choose it to be F itself which is the lowest order term in the expansion of $\mathcal{M}$. Then in averaging over $P[\zeta(\underset{\sim}{u})]$ one needs quantities like

$$\left\langle \zeta(\underset{\sim}{x}_1)\ldots\zeta(\underset{\sim}{x}_n)\, e^{-iQ_3\zeta(\underset{\sim}{x})}\, e^{iK_3\zeta(\underset{\sim}{x}_o)} \right\rangle . \tag{50}$$

These can be evaluated from $Z[J]$ as

$$\left.\frac{\partial^n Z[J(\underset{\sim}{u})]}{\partial J(\underset{\sim}{x}_1)\ldots\partial J(\underset{\sim}{x}_n)}\right|_{J(\underset{\sim}{u}) = -iQ_3\delta(\underset{\sim}{u}-\underset{\sim}{x}) + iK_3\delta(\underset{\sim}{u}-\underset{\sim}{x}_o)} \tag{51}$$

A final note on these developments. In first separating out the fluctuating sources, we have decoupled the problem of evaluating operators like $\mathcal{M}$ (which really means $(1-G_oF)^{-1}$) and the problem of averaging over ζ with moving sources. The evaluation of $\mathcal{M}$ by approximate means; e.g., a parabolic approximation to $G_o^{-1}-F$ and a formal solution to that approximation, allows that approximation to be put in (39) and directly averaged over ζ .

III A RANDOM SURFACE AND A FIXED BOTTOM

We remain in the slow surface approximation but now imagine the source and receiver to lie in the slice $\zeta(\underset{\sim}{x}) \leq z \leq B$ where $z=B$ is the bottom and $\zeta(\underset{\sim}{x})$ the sea surface. We do not want to simply make a change of variables that will flatten out the surface since the construction of G_o by the image method will not lead to a closed solution. So instead we set

$$\xi = \frac{z-\zeta(\underset{\sim}{x})}{B-z} \tag{52}$$

and

$$\underset{\sim}{\rho} = \underset{\sim}{x} \quad ,$$

which flattens the sea surface to $\xi=0$ and sends the bottom to $\xi=\infty$. If we choose as boundary conditions on the pressure $\psi(z,x)$

$$\psi\left(\zeta(\underset{\sim}{x}),\ \underset{\sim}{x}\right) = 0 \tag{53}$$

and

$$\psi(B,\underset{\sim}{x}) = 0 \quad , \tag{54}$$

our mapped pressure $\Psi(\xi,\underset{\sim}{\rho})$ satisfies

$$\Psi(0,\underset{\sim}{\rho}) = 0 \tag{55}$$

$$\Psi(\xi,\rho) \to 0 \text{ as } \xi^2 + \rho^2 \to 0 \quad . \tag{56}$$

These are precisely the boundary conditions we had before. Indeed, the only difference now is in the fluctuation operator, for the wave equation reads

$$\left[\frac{\partial^2}{\partial\xi^2} + \nabla_{\perp}^2 + k^2 - F_B(\xi,\rho)\right]\Psi(\xi,\rho) = \delta^2(\rho-\rho_o) \times \delta\left(\xi - \frac{z_o-\zeta(\rho_o)}{B-z_o}\right) \quad , \tag{57}$$

where

$$F_B(\xi,\rho) = \left[\nabla_{\perp}^2\zeta - \frac{2}{B-\zeta}\left((\partial_j\zeta)^2 + (1+\xi)^2\right)\right]\frac{1+\xi}{B-\zeta}\frac{\partial}{\partial\xi} + 2\frac{\partial\zeta}{\partial\rho_j}\frac{1+\xi}{B-\zeta}\frac{\partial^2}{\partial\rho_j\partial\xi} + \left[1-(1+\xi)^2-(\partial_j\zeta)^2\right]\frac{(1+\xi)^2}{(B-\zeta)^2}\frac{\partial^2}{\partial\xi^2} \quad . \tag{58}$$

The fluctuation operator is clearly more involved than in the no bottom problem addressed above.

If we define the operator G_B in direct analogy to the previous case,

then

$$\Psi = G_B \, S_B \tag{59}$$

with

$$G_B = \frac{1}{1-G_o F_B} G_o = G_o \frac{1}{1-F_B G_o} , \tag{60}$$

and

$$S_B(\xi,\rho) = \delta^2(\rho-\rho_o)\delta\left(\xi - \frac{z_o-\zeta(\rho_o)}{B-z_o}\right) , \tag{61}$$

so

$$\psi(z,\underset{\sim}{x}) = G_B\left(\frac{z-\zeta(\underset{\sim}{x})}{B-z}, \underset{\sim}{x}; \frac{z_o-\zeta(\underset{\sim}{x}_o)}{B-z_o}, \underset{\sim}{x}_o\right) . \tag{62}$$

Our reward for facing up to the complicated fluctuation operator (58) is that we may now use precisely the same $G_o(\xi,\rho;\ \xi',\ \rho')$ as given before in Eq. (24). This is because we have mapped the boundaries $z=\zeta$ and $z=B$ into $\xi=0$ and $\xi=\infty$ as before and have the same boundary conditions now. A concrete formula such as (48) or (49) may be given for the first term in the series in F_B for G_B, and, indeed the discussion goes as before for the methods of evaluation of non-perturbative approximations to G_B beginning with

$$G_B = G_o + G_o M_B G_o , \tag{63}$$

and

$$M_B = \frac{1}{2}\left(F_B \frac{1}{1-G_o F_B} + \frac{1}{1-F_B G_o} F_B\right) . \tag{64}$$

Just as a side note, the case when $\zeta(\underset{\sim}{x})$ is constant, say $\zeta=0$, is not soluble in closed form by the image charge method[6]. However, the mapping we have introduced allows us to construct a Green function for the Helmholtz equation with a point source between infinite plates at $z=0$ and $z=B$ as a series in the operator

$$F_B(\zeta=0) = -\frac{2(1+\xi)^3}{B^2}\frac{\partial}{\partial\xi} - \frac{\xi(2+\xi)(1+\xi)^2}{B^2}\frac{\partial^2}{\partial\xi^2} \tag{65}$$

Each term of the series obeys the correct boundary conditions (53) and (54). So this observation may prove useful in its own right.

IV THE TIME DEPENDENT RANDOM SURFACE

Now we apply our mapping to the time dependent random surface. First we consider the source to lie in $\zeta(\underset{\sim}{x},t) \leq z \leq \infty$. This is the situation indicated in Figure 1 and described in Eqs. (7) and (8) before.

As is by now familiar, we set

$$\xi = z - \zeta(\underset{\sim}{x},t) \quad , \tag{66}$$

$$\rho = \underset{\sim}{x} \quad , \tag{67}$$

and

$$\tau = t \quad . \tag{68}$$

The wave equation becomes now

$$\left[\frac{\partial}{\partial\xi^2} + \nabla_\perp^2 - \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2} - F_T(\xi,\rho,\tau)\right]\Psi(\xi,\rho,\tau) =$$

$$S(\tau)\delta^2(\rho-\rho_o)\delta\left(\xi-\left(z-\zeta\left(\rho_o,\tau\right)\right)\right) \tag{69}$$

and the boundary conditions are

$$\Psi(0,\rho,\tau) = 0 \quad , \tag{70}$$

and

$$\Psi(\xi,\rho,\tau) \to 0 \text{ as } \xi^2 + \rho^2 \to \infty \quad . \tag{71}$$

The fluctuation operator for the time dependent case reads

$$F_T(\xi,\rho,\tau) = \left[\frac{1}{c^2}\left(\frac{\partial\zeta}{\partial\tau}\right)^2 - (\nabla_\perp\zeta)^2\right]\frac{\partial^2}{\partial\xi} - \frac{2}{c^2}\frac{\partial\zeta}{\partial\tau}\frac{\partial^2}{\partial\xi\partial\tau}$$

$$+ (\nabla_\perp^2\zeta)\frac{\partial}{\partial\xi} + 2\frac{\partial\zeta}{\partial\rho_j}\frac{\partial^2}{\partial\rho_j\partial\xi} \quad . \tag{72}$$

Calling

$$\mathcal{S}_T(\xi,\underset{\sim}{\rho},\tau) = F_T(\xi,\underset{\sim}{\rho},\tau)\Psi(\xi,\underset{\sim}{\rho},\tau) + S(\tau)\delta^2(\underset{\sim}{\rho}-\underset{\sim}{\rho}_o)\delta\Big(\xi-\big(z_o-\zeta(\underset{\sim}{\rho}_o,\tau)\big)\Big) \quad , \tag{73}$$

the solution to (69) is given in terms of the usual retarded Green function satisfying

$$\left(\frac{\partial^2}{\partial\xi^2} + \nabla_\perp^2 - \frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}\right)G_o(\xi,\underset{\sim}{\rho},\tau;\ \xi',\underset{\sim}{\rho}',\tau') = \delta(\xi-\xi')\delta^2(\underset{\sim}{\rho}-\underset{\sim}{\rho}')\delta(\tau-\tau'), \tag{74}$$

and for $\Psi(\xi,\underset{\sim}{\rho},\tau)$ we have[7]

$$\Psi(\xi,\rho,\tau) = \int_{\tau_o}^{\tau+\varepsilon} d\tau' d\xi' d^2\rho' G_o(\xi,\rho,\tau;\ \xi',\rho',\tau')\mathcal{J}_T(\xi',\rho',\tau')$$

$$+ \int_{\tau_o}^{\tau_o+\varepsilon} d\tau' \int d^2\rho' \Bigg\{ G_o(\xi,\rho,\tau;\ \xi',\rho',\tau') \frac{\partial}{\partial\xi'} \Psi(\xi',\rho',\tau')$$

$$- \frac{\partial}{\partial\xi'} G_o(\xi,\rho,\tau;\ \xi',\rho',\tau')\Psi(\xi',\rho',\tau')\Bigg\}\Bigg|_{\xi'=0}$$

$$+ \frac{1}{c^2} \int d\xi' d^2\rho' \left[\frac{\partial G_o}{\partial\tau'} (\xi,\rho,\tau;\ \xi',\rho',\tau')\Psi(\xi',\rho',\tau')\right.$$

$$\left. - G_o(\xi,\rho,\tau;\ \xi'\rho'\tau')\partial/\partial\tau'\Psi(\xi',\rho',\tau')\right]_{\tau'=\tau_o} , \qquad (75)$$

where $\varepsilon \to 0+$ after all integrations. We have used the boundary condition on Ψ at spatial infinity and assumed G_o vanishes there as well.

Now let us imagine that we turned the source on at some finite time and let $\tau_o \to -\infty$ where we take Ψ and $\partial\Psi/\partial\tau = 0$. We can choose G_o to vanish at $\xi = 0$ by our image method as before. The correct G_o which vanishes for $\tau<\tau'$ is

$$G_o(\xi,\underset{\sim}{\rho},\tau;\ \xi',\underset{\sim}{\rho}',\ \tau') = \int \frac{d^4Q}{(2\pi)^4} \frac{e^{-iQ_o(\tau-\tau')}\ e^{i\underset{\sim}{Q}\cdot(\underset{\sim}{\rho}-\underset{\sim}{\rho}')}}{\left(\frac{Q_o+i\varepsilon}{c}\right)^2-\underset{\sim}{Q}^2-Q_3^2}\ \times$$

$$\left\{e^{iQ_3(\xi-\xi')}-e^{iQ_3(\xi+\xi')}\right\} \quad . \tag{76}$$

With these choices the pressure at time τ satisfies the integral equation

$$\Psi(\xi,\underset{\sim}{\rho},\tau)=\int_{-\infty}^{\infty} d\tau' \int_{0}^{\infty} d\xi' \int d^2\rho' G_o\left(\xi,\underset{\sim}{\rho},\tau;\ \xi',\underset{\sim}{\rho}',\tau'\right) S_T(\xi',\underset{\sim}{\rho}',\tau')$$

$$+ \int_{-\infty}^{\infty} d\tau' \int_{0}^{\infty} d\xi' \int d^2\rho' G_o(\xi,\rho,\tau;\ \xi',\rho',\tau')F_T(\xi',\rho',\tau')\Psi(\xi',\rho',\tau') \quad ,\tag{77}$$

and this has the by now familiar formal solution

$$\Psi = G_T\ S_T \tag{78}$$

with

$$G_T = \frac{1}{1-G_oF_T}\ G_o - G_o\ \frac{1}{1-F_TG_o} \tag{79}$$

and

$$S_T(\xi,\underset{\sim}{\rho},\tau) = S(\tau)\delta^2(\underset{\sim}{\rho}-\underset{\sim}{\rho}_0)\delta\left(\xi-\left(z_0-\zeta\left(\underset{\sim}{\rho}_0,\tau\right)\right)\right) \quad . \tag{80}$$

The pressure field we seek is

$$\psi(z,\underset{\sim}{x},\tau) = \int S(t_0)dt_0 G_T\left(z-\zeta\left(\underset{\sim}{x},t\right),\underset{\sim}{x},t;z_0-\zeta\left(\underset{\sim}{x}_0 t_0\right),\underset{\sim}{x}_0,t_0\right) \quad , \tag{81}$$

so the issue again is the construction of the complete Green function G_T .

Using (76) it is straightforward to give an integral representation for $\left\langle G_0 \right\rangle$, the first term in the series in F_T for the construction of G_T . When $\zeta(\underset{\sim}{x},t)$ is a gaussian random function with zero mean and correlation function

$$\left\langle \zeta(\underset{\sim}{u},t)\zeta(\underset{\sim}{w},t') \right\rangle = \Gamma(\underset{\sim}{u}-\underset{\sim}{w},t-t') \quad , \tag{82}$$

expressions similar to (48) and (49) emerge. We do not record them here.

With all this work done, it is easy to state what one does when the ocean has a bottom at $z = B$. Clearly one goes over to the usual

$$\xi = \frac{z-\zeta(\underset{\sim}{x},t)}{B-z}, \qquad \rho = \underset{\sim}{x}, \; \tau = t \tag{83}$$

and after identifying the fluctuation operator F_{TB} for this case and the full $G_{TB} = (1-G_o F_{TB})^{-1} G_o$, we will have

$$\psi(z,\underset{\sim}{x},t) = \int_{-\infty}^{+\infty} dt_o S(t_o) G_{TB}\left(\frac{z-\zeta(\underset{\sim}{x},t)}{B-z}, \underset{\sim}{x}, t; \frac{z_o-\zeta(\underset{\sim}{x}_o,t_o)}{B-z_o}, \underset{\sim}{x}_o, t_o\right) \quad , \qquad (84)$$

as usual.

All of the previous remarks on expanding the full Green function in a series in the fluctuation operator apply to expansion of G_{TB} in F_{TB}, so no more will be said here in that regard.

V SUMMARY

This paper has employed a simple device to recast the problem of scalar wave propagation in the presence of a random surface into an equivalent, equally difficult, but perhaps more tractable problem of wave propagation in a random medium with randomly moving source and receiver. We have taken the original geometry, typically a source and receiver between a random surface and a fixed bottom, and mapped it into a geometry of a source and receiver between a fixed plane and the surface at infinity above it. In this operation the wave operator $\nabla^2 - c^{-2}\partial_t^2$ in the old co-ordinates goes over into the wave operator in the new co-ordinates plus a fluctuation operator. Any boundary conditions on the original scalar field can now be exactly met, order by order in perturbation theory in the fluctuation operator, since the Green function for the simple geometry is available.

In the new co-ordinate system the problem suggests a variety of novel approximation procedures for the construction of the received field. The form: full wave operator = usual wave operator + fluctuation operator is familiar from quantum mechanical scattering in a potential and also from quantum field theory[3,4]. Using some idea like an eikonal or Rytov approximation for the full wave operator will provide a rich set of expressions for the scattered field, and in the examples

considered here allow one to carry out explicitly the required averaging over the random surface when that surface has a gaussian distribution.

We considered here only acoustic waves propagating in a region with a random surface at which surface the pressure is to vanish. Since the mapping is purely geometrical, it clearly will apply to electromagnetic waves scattering from a random surface and to either scalar or electromagnetic waves with more diverse boundary conditions. Some of these topics are under investigation now.

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